

A LECTURE ON NEUTRINOS ¹

S. M. Bilenky

Joint Institute for Nuclear Research, Dubna, R-141980, Russia

SISSA, via Beirut 2-4, Trieste, 34014, Italy

Abstract

The major steps in the development of our knowledge about neutrinos are reviewed. The basics of neutrino oscillation formalism is presented. Neutrino oscillations in the framework of three-neutrino mixing are considered. The evidence for neutrino oscillations is discussed.

1 Introduction

Neutrinos played a key role in the establishment of the unified theory of the electromagnetic and weak interactions, the Standard Model (SM). The first evidence for the unification (the discovery of Neutral Currents) was obtained in the neutrino experiments at CERN in 1973. The measurement of the number of the flavor neutrinos in the LEP experiments in the nineties allowed to determine the number of quark-lepton families etc.

After the SM was fully established in high precision LEP and other experiments, the most important discovery in elementary particle physics was the recent evidence found in the Super-Kamiokande [1], SNO [2], KamLAND [3] and other solar [4, 5, 6, 7] and atmospheric [8, 9] neutrino experiments *of a new phenomenon, neutrino oscillations*.

Observation of neutrino oscillations means that flavor lepton numbers L_e , L_μ and L_τ are not conserved by a neutrino mass term of the Lagrangian, which is the source of neutrino masses and mixing. Small neutrino masses and neutrino mixing are apparently generated by *a new, beyond the SM mechanism*. Many further investigations must be performed, however, in order to reveal the true nature of the discovered phenomenon.

The history of neutrinos is very interesting. One of its striking feature is that the most fundamental and pioneer ideas were only partially correct. Great courage and great intuition was required to propose them.

¹A talk at the colloquium at the University of Torino in December 2003.

2 Some historical remarks

Pauli assumed the existence of neutrino in 1930 in order to explain the continuous β - spectrum. He suggested the existence of a neutral, spin 1/2 particle with mass of the same order of magnitude as the electron mass and penetration length much larger than that of the photon (Pauli called this new particle neutron). Pauli assumed that in the β -decay of nuclei a neutrino is emitted together with the electron

$$(A, Z) \rightarrow (A, Z + 1) + e^- + \nu.$$

The released energy will be shared in this case by the electron and the neutrino (which can not be detected in the β -decay experiments) and a continuous spectrum of electrons is observed. In spite of the fact this is the only way to explain continuous β -spectra in the framework of the conservation of the total energy and momentum, *in the thirties the very idea of a new particle was very courageous*.²

Pauli presented his idea of neutrino in the famous letter addressed to the participants of the Tübingen conference. In the same letter Pauli assumed, however, that neutrinos are constituents of nuclei.

At that time nuclei were considered as bound states of protons and electrons. This assumption was in a contradiction with the theorem on the connection between spin and statistics. For example, ${}^7\text{N}_{14}$ nucleus according to the electron-proton model must have half-integer spin (14 protons and 7 electrons). From the experimental data it was known, however, that ${}^7\text{N}_{14}$ nuclei satisfy Bose-Einstein statistics. Pauli thought that additional spin 1/2 neutral particles in nuclei will be a remedy. After the discovery of the neutron by J. Chadwick in 1932 [14] the electron-proton model and the problem of spin of ${}^7\text{N}_{14}$ disappeared.

Fermi was the first who understood that the electron-neutrino pair is *produced in the transition*:³

²It is well known that Dirac in the paper “A theory of electrons and protons” [10], in which the idea of the electron sea was presented for the first time, identified the holes in the sea with protons. Later he explained that “the whole climate of opinion at that time was against new particles”(see [11]) . In 1931 Dirac came to the conclusion that the hole must be a new particle with positive charge and a mass equal to the electron mass [12]. Dirac predicted the positron before it was discovered by Anderson in 1932 [13]

³It took about one year for Fermi to realize how electron and neutrino (which are not constituent of nuclei) are produced. Even though quantum field theory existed the idea of

$$n \rightarrow p + e^- + \bar{\nu}. \quad (1)$$

The effective Hamiltonian of the decay (1), proposed by Fermi, had the form

$$\mathcal{H}_I^\beta = G_F \bar{p} \gamma_\alpha n \bar{e} \gamma^\alpha \nu + \text{h.c.}, \quad (2)$$

where G_F is the interaction constant (Fermi constant). By analogy with electromagnetic interaction

$$\mathcal{H}_I^{\text{em}} = e \bar{p} \gamma_\alpha p A^\alpha \quad (3)$$

Fermi assumed that β -decay interaction is a vector interaction.

The Hamiltonian (2) describes allowed β -decay transitions in which spins and parities of the initial and final nuclei are the same (Fermi selection rule):

$$\Delta I = 0 \quad \pi_i = \pi_f.$$

This Hamiltonian cannot describe, however, decays, which satisfy Gamov-Teller selection rule:

$$\Delta I = \pm 1, 0 \quad \pi_i = \pi_f.$$

Thus, it was clear in the thirties that the analogy between β -decay and electromagnetic interactions can be only partially correct.

The most general four-fermion Hamiltonian of the β -decay was proposed by Gamov and Teller in 1936. It has the form of the sum of the products of scalars (S), vectors (V), tensors (T), axial vectors (A) and pseudoscalars (P):

$$\mathcal{H}_I^\beta = \sum_{i=S,V,T,A,P} G_i \bar{p} O_i n \bar{e} O^i \nu + \text{h.c.} \quad (4)$$

Here

$$O_i \rightarrow 1, \gamma_\alpha, \sigma_{\alpha\beta}, \gamma_\alpha \gamma_5, \gamma_5.$$

and G_i are constants.

creation of new particles in transitions of known ones, was not so evident at that time as now. For Fermi analogy with emission of photons in electromagnetic transitions was very important (P.Pontecorvo, private communication).

For many years the main aim of β -decay experiments was the search for the dominant terms in the Hamiltonian (4). Up to 1956 the situation with the Hamiltonian of the β -decay was unclear and contradictory.

In 1956 the violation of parity in the β -decay was discovered in the Wu et al. [15] and other experiments. In the Wu et al. experiment β -decay of the polarized ^{60}Co was studied. In the general case of the non conservation of parity the probability of the emission of the electron in the β -decay of a nucleus with the polarization \vec{P} is given by the following expression

$$w_{\vec{P}}(\vec{k}) = w_0 (1 + \alpha \vec{P} \cdot \vec{k}) = w_0 (1 + \alpha P \cos \theta), \quad (5)$$

where $\vec{k} = \vec{p}/p$ is the unit vector in the direction of the momentum of the electron. If parity is conserved

$$w_{\vec{P}}(\vec{k}) = w_{\vec{P}}(-\vec{k})$$

and $\alpha = 0$ (the pseudoscalar $\vec{P} \cdot \vec{k}$ can enter into the expression for the decay probability if the parity is conserved). In the Wu et al experiment it was found that $\alpha \simeq -0.7$. Thus, large violation of parity in the β -decay was discovered in the experiment.

The violation of parity in the β -decay means that the Hamiltonian of the process is a sum of scalar and pseudoscalar. The first Hamiltonian of this type was proposed by Lee and Yang in 1956 [16] before the Wu et al. experiment has been done. The Hamiltonian, they considered, was a direct generalization of the Fermi-Gamov-Teller Hamiltonian (4) and had the form

$$\mathcal{H}_I^\beta = \sum_{i=S,V,T,A,P} \bar{p} O_i n \bar{e} O^i (G_i - G'_i \gamma_5) \nu + \text{h.c.}, \quad (6)$$

where the constants G'_i characterize the pseudoscalar part of the Hamiltonian. In the Hamiltonian (6) ten (!) arbitrary constants enter.

The major progress in the establishment of the true effective Hamiltonian of the weak interaction was reached in 1957-58. Two decisive steps were done at that time. *The first step was connected with neutrino.*

In 1957 Landau [17], Lee and Yang [18] and Salam [19] proposed *the theory of the two-component neutrino*. Pauli considered the neutrino a particle with a mass different from zero and of the order of the electron mass m_e . At the time of the discovery of the parity violation from the experiments on the

measurement of the high-energy part of the β - spectrum of 3H , it was found that neutrino mass m is much less than the mass of the electron:

$$m \lesssim 200 \text{ eV} \ll m_e.$$

The authors of the two-component neutrino theory assumed that *neutrino is massless particle*. For massless neutrino exists a very attractive possibility for large violation of the parity in the processes of emission (and absorption) of neutrino.

The field of neutrino $\nu(x)$ in the general case $m \neq 0$ satisfies the Dirac equation

$$(i \gamma^\alpha \partial_\alpha - m) \nu(x) = 0. \quad (7)$$

For the left-handed and right-handed components

$$\nu_{L,R}(x) = \frac{1 \mp \gamma_5}{2} \nu(x), \quad (8)$$

from (7) we have *the system of two coupled equations*

$$i \gamma^\alpha \partial_\alpha \nu_L - m \nu_R = 0; \quad i \gamma^\alpha \partial_\alpha \nu_R - m \nu_L = 0. \quad (9)$$

If $m = 0$ from (9) we obtain *the decoupled Weyl equations*:

$$i \gamma^\alpha \partial_\alpha \nu_{L,R}(x) = 0. \quad (10)$$

Thus, in the case of the massless neutrino for neutrino field $\nu_L(x)$ (or $\nu_R(x)$) can be chosen. ⁴ This was the choice, made by Landau, Lee and Yang and Salam.

If the neutrino field is $\nu_L(x)$ ($\nu_R(x)$) :

1. Parity is strongly violated :

$$G'_i = G_i \quad (G'_i = -G_i). \quad (11)$$

2. The helicity of neutrino is equal to 1 (-1) and helicity of antineutrino is equal to -1 (1).

⁴The equations (10) for $\nu_L(x)$ and $\nu_R(x)$ were discussed by Pauli in his famous encyclopedia article "General Principles of Quantum Mechanics" [20]. Pauli stressed that these equations are not invariant under space reflections and "therefore are not applicable to the physical reality."

The helicity of neutrino was measured in the famous Goldhaber et al. experiment in 1958 [21]. In this experiment the circular polarization of γ -quanta produced in the process

$$\begin{array}{ccc} e^- + \text{Eu} \rightarrow \nu_e + & \text{Sm}^* & \\ & \downarrow & \\ & \text{Sm} & + \gamma \end{array} \quad (12)$$

was measured. The measurement of the polarization of the γ -quanta allows to determine the helicity of neutrino. It was found that

$$h = -1 \pm 0.3 \quad (13)$$

Thus, the result of the Goldhaber et al. experiment was in agreement with the two-component neutrino theory. It was proved that of the two possibilities $\nu_L(x)$ or $\nu_R(x)$, the first one is realized.⁵

The next decisive step in the creation of the effective Hamiltonian of the weak interaction was done by Feynman and Gell-Mann [22], Marshak and Sudarshan [23]. These authors assumed that not only neutrino field but *all fermion fields* enter in the Hamiltonian of the weak interaction in the form of the left-handed components.⁶

If only left-handed components of the fermion fields enter into the Hamiltonian, the most general four-fermion Hamiltonian of the β -decay is characterized by only one constant and has the form

$$\mathcal{H}_I^\beta = \frac{G_F}{\sqrt{2}} 4 \bar{p}_L \gamma_\alpha n_L \bar{e}_L \gamma^\alpha \nu_L + \text{h.c.} \quad (14)$$

The theory with the Hamiltonian (14) was very successful: it allowed to describe the data of all β -decay experiments.

Feynman and Gell-Mann introduced *the very important notion of charged weak current*

$$j^\alpha = 2 [\bar{p}_L \gamma^\alpha n_L + \bar{\nu}_{eL} \gamma^\alpha e_L + \bar{\nu}_{\mu L} \gamma^\alpha \mu_L]. \quad (15)$$

⁵We know today that neutrino masses are different from zero. However, due to the smallness of neutrino masses, two-component theory is an extremely good approximation.

⁶Let us notice that this was a courageous proposal: at the time when it was done there were some β -decay experiments, which were in contradiction with assumption made in [22, 23]. Later it occurred that the experiments were wrong.

and assumed that the Hamiltonian of the weak interaction has the current \times current, $V - A$ form

$$\mathcal{H}_I = \frac{G_F}{\sqrt{2}} j^\alpha j_\alpha^+. \quad (16)$$

This Hamiltonian allowed to describe not only β -decay but also μ -capture, μ -decay and other weak processes.

In the Feynman and Gell-Mann paper [22], a possible origin of the current \times current interaction was discussed. They considered the weak interaction theory with the intermediate heavy vector W^\pm boson. If we assume that the true Lagrangian of the weak interaction has the form

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} j_\alpha W^\alpha + \text{h.c.}, \quad (17)$$

where g is the dimensionless interaction constant, the effective Hamiltonian of the processes with the virtual W boson with momentum Q , which satisfies the condition $Q^2 \ll m_W^2$, has the current \times current form (16). The Fermi constant is connected with the constant g by the relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}. \quad (18)$$

With the further development of the theory of the weak interaction the notion of the charged weak current drastically changed. In 1962 in the Brookhaven experiment [24] it was shown that electron neutrino ν_e and muon neutrino ν_μ are different particles.⁷

In 1962 strange particles were included into the weak charged current by N. Cabibbo [26]. Later with the establishment of the notion of quarks, fields of nucleons in the weak current were replaced by the fields of quarks. In 1970 after the Glashow-Iliopoulos-Maiani paper [27], the charged weak current took the form

$$j_\alpha = 2 [\bar{\nu}_{eL} \gamma_\alpha e_L + \bar{\nu}_{\mu L} \gamma_\alpha \mu_L + \bar{u}_L \gamma_\alpha d'_L + \bar{c}_L \gamma_\alpha s'_L], \quad (19)$$

where $c(x)$ is the field of the *predicted* charmed quark and d'_L and s'_L are orthogonal combinations (mixture) of the d_L and s_L fields

$$\begin{aligned} d'_L &= \cos \theta_C d_L + \sin \theta_C s_L \\ s'_L &= -\sin \theta_C d_L + \cos \theta_C s_L, \end{aligned} \quad (20)$$

⁷The idea of this experiment was proposed by B. Pontecorvo in 1959 [25]

where θ_C is the Cabibbo angle.

First charmed particles were successfully discovered in 1975. The data on the investigation of the decays of strange and charmed particles and neutrino processes were in good agreement with the mixing relations (20).

After the establishment of the quark mixing (20) it was natural to assume that quark-lepton analogy holds: neutrino masses like masses of all other fundamental fermions (quarks and leptons) are different from zero and fields of massive neutrinos enter into charged current in a mixed form

$$\begin{aligned}\nu_{eL} &= \cos \theta \nu_{1L} + \sin \theta \nu_{2L} \\ \nu_{\mu L} &= -\sin \theta \nu_{1L} + \cos \theta \nu_{2L},\end{aligned}\tag{21}$$

where θ is a lepton mixing angle. Neutrino oscillations and other consequences of the neutrino mixing (21) were considered in the seventies (see [28]).

Up to now we have discussed the phenomenological period of the development of the theory of the weak interaction. The real theory, the Standard Model, was created in the seventies (Glashow [29], Weinberg [30], Salam [31]). This theory was born in an attempt to construct a renormalizable theory of the weak interaction. The only physical renormalizable theory, known at that time, was quantum electrodynamics. The success in the creation of the renormalizable theory of the weak interaction was reached on the way of the *unification* of the weak and electromagnetic interactions into one electroweak interaction.

The SM is based on $SU(2) \times U(1)$ local gauge invariance and the Higgs mechanism of the spontaneous violation of symmetry. The SM summarizes the development of elementary particle physics in the last century and is one of the greatest achievement in physics. The SM Lagrangian of the interaction of fermions and vector bosons has the form of the sum of the charged current (CC), neutral current (NC) and electromagnetic terms:

$$\mathcal{L}_I = \left(-\frac{g}{2\sqrt{2}}j_\alpha^{CC}W^\alpha + h.c.\right) - \frac{g}{2\cos\theta_W}j_\alpha^{NC}Z^\alpha - ej_\alpha^{\text{em}}A^\alpha.\tag{22}$$

Here j_α^{CC} , j_α^{NC} and j_α^{em} are charged, neutral and electromagnetic currents, W^α and Z^α are fields of W^\pm and Z^0 vector bosons, A^α is the electromagnetic field and θ_W is the weak angle (parameter of the theory).

The SM *predicted* the existence of W^\pm and Z^0 vector bosons and their masses, *predicted* new NC interaction, *predicted* charmed, bottom, top and

other particles etc. All prediction of the SM are in perfect agreement with the data of many experiments, including very precise LEP experiments.⁸

The SM *provides a natural framework for the quark mixing* After spontaneous violation of symmetry for the quark charged current we have

$$j_\alpha^{CC} = 2[\bar{u}_L \gamma_\alpha d'_L + \bar{c}_L \gamma_\alpha s'_L + \bar{t}_L \gamma_\alpha b'_L], \quad (23)$$

where

$$d'_L = \sum_{q=d,s,b} V_{uq} q_L; \quad s'_L = \sum_{q=d,s,b} V_{cq} q_L; \quad b'_L = \sum_{q=d,s,b} V_{tq} q_L. \quad (24)$$

Here V is the unitary Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, which is characterized by three mixing angles and one CP phase [26, 32].

In 1967 when Weinberg and Salam papers appeared only two charged leptons e and μ and two neutrinos ν_e and ν_μ were known. After the success of the theory of the two-component neutrino there was at that time general opinion that neutrinos were massless particles. The original SM was build for two charged leptons and *two massless neutrinos*.

Neutrino masses and mixing can be introduced, however, in the framework of the SM with the Higgs doublet in the same manner as masses and mixing of quarks are introduced. Neutrino fields ν_{lL} ($l = e, \mu, \tau$) that enter in this case into the lepton charged current

$$j_\alpha^{CC} = 2[\bar{\nu}_{eL} \gamma_\alpha e_L + \bar{\nu}_{\mu L} \gamma_\alpha \mu_L + \bar{\nu}_{\tau L} \gamma_\alpha \tau_L], \quad (25)$$

are mixtures of the fields of neutrinos with definite masses

$$\nu_{lL} = \sum_{i=1}^3 U_{li} \nu_{iL} \quad (26)$$

where ν_i is the field of neutrino with the mass m_i and U is the unitary mixing matrix. Like the CKM matrix the matrix U is characterized by three mixing angles and one CP phase.

⁸In spite of this impressive agreement with the existing data, the SM, as it is well known, can not be considered as a satisfactory theory. There are several open problems in the SM. They are mainly connected with the Higgs mechanism of the spontaneous violation of the symmetry: hierarchy problem, the problem of the large number of free parameters (fermion masses, mixing angles, CP-phases) etc.

We will come back to the relation (26) later. Now we will discuss the *birth of the idea of neutrino mixing and oscillations*. This idea was proposed by B. Pontecorvo in 1957-58 [33, 34] soon after the two-component neutrino theory appeared. At that time only the electron neutrino was known.

B. Pontecorvo was fascinated by the $K^0 \rightleftharpoons \overline{K}^0$ oscillations and looked for the analogous phenomenon in the lepton world. For the first time he mentioned a possibility of transitions of $\nu \rightleftharpoons \bar{\nu}$ in vacuum in 1957 in the paper [33], in which the transitions of muonium into antimuonium ($(\mu^+e^-) \rightleftharpoons (\mu^-e^+)$) were discussed. In this paper he wrote

“If the two-component neutrino theory turn out to be incorrect (which at present seems to be rather improbable) and if the conservation law of neutrino charge would not apply, then in principle neutrino \rightleftharpoons antineutrino transitions could take place in vacuum.”

In the fifties Reines and Cowan have been doing their famous experiment [35] in which $\bar{\nu}_e$'s from a reactor were discovered via the observation of e^+ and n in the process

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

In 1957-58 another experiment with reactor antineutrinos was done by R. Davis [36]. In this experiment the process

$$\bar{\nu}_e + {}^{37}\text{Cl} \rightarrow e^- + {}^{37}\text{Ar}, \quad (27)$$

in which the lepton number is violated, was searched for. A rumor reached B. Pontecorvo that R. Davis observed events (27). He came to the conclusion that neutrino oscillations could be a possible and natural explanation of the “observed events” and published the first paper on neutrino oscillations [34].

B. Pontecorvo assumed that neutrino and antineutrino, produced in usual weak processes, are different particles and there exists an additional interaction, which transfer neutrino into antineutrino. He concluded that in this case “neutrino and antineutrino are *mixed* particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity”⁹.

There were two parts in the Pontecorvo paper [34]. First he suggested that in the Reines and Cowan experiment

⁹In the first papers on neutrino oscillations B. Pontecorvo considered maximum mixing

“The cross section of the production of neutrons and positrons in the process of the absorption of antineutrinos from a reactor by protons would be smaller than the expected cross section....It would be extremely interesting to perform the Reines and Cowan experiment at different distances from reactor.”

In 2002 the effect, predicted by B. Pontecorvo in 1958, was observed in the reactor KamLAND experiment [3], in which the average distance between reactors and the detector is about 180 km. Of course, B. Pontecorvo could not know the distance at which the effect can be observed. He noticed in [34]

“Effects of transformation of neutrino into antineutrino and vice versa may be unobservable in the laboratory because of the large values of R , but will certainly occur on an astronomical scale.”

In the second part of the paper [34] B. Pontecorvo tried to explain “events” (27). He wrote

“It is not possible to state a priori that antineutrino flux, which at the beginning can not initiate the reaction (27), is not transferred into a flux some part of which could produce this reaction”

In the framework of the two-component theory right-handed antineutrinos, produced in decays of neutron-rich nuclei in a reactor, can not induce the reaction (27). Later, when the two-component neutrino theory was established B. Pontecorvo understood that *in the case of one neutrino* right-handed antineutrino $\bar{\nu}_e$ can transfer *only* into right-handed neutrino ν_{eR} , which is *a sterile particle*. In fact, B. Pontecorvo was the first who introduced [37] the notion of sterile neutrinos so popular nowadays.

After the second neutrino ν_μ was discovered in the Brookhaven experiment it was not difficult for B. Pontecorvo to generalize his idea of neutrino oscillations for the case of two types of neutrinos [37]. In 1967 before the first results of the R. Davis solar neutrino experiment were published he *envisaged the solar neutrino problem*:

“From an observational point of view the ideal object is the sun. If the oscillation length is smaller than the radius of the sun region effectively producing neutrinos, (let us say one tenth of the sun radius R_\odot or 0.1 million km for 8B neutrinos, which will give the

main contribution in the experiments being planned now), direct oscillations will be smeared out and unobservable. The only effect on the earth's surface would be that the flux of observable sun neutrinos must be two times smaller than the total (active and sterile) neutrino flux."

In 2002-03 this prediction was confirmed in a model-independent way by the SNO solar neutrino experiment [2].

With these remarks we conclude the part connected with the history of neutrino and we come to the discussion of the present status of neutrino oscillations.

3 Neutrino oscillations

In all solar [2, 4, 5, 6, 7] and atmospheric [1, 8, 9] and in the reactor KamLAND [3] neutrino experiments *compelling evidence of neutrino oscillations was obtained*. The interpretation of the data of these experiments is based on the following assumptions:

1. The Lagrangians of the interaction of neutrinos with other particles are the standard CC and NC Lagrangians (22) with the lepton charged current and neutrino neutral current given by the expressions

$$j_{\alpha}^{\text{CC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_{\alpha} l_L; \quad j_{\alpha}^{\text{NC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_{\alpha} \nu_{lL}. \quad (28)$$

2. Tree flavor neutrinos $\nu_e, \nu_{\mu}, \nu_{\tau}$ (and antineutrinos $\bar{\nu}_e, \bar{\nu}_{\mu}, \bar{\nu}_{\tau}$) exist in nature.
3. Neutrino mixing takes place:

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}, \quad (29)$$

where ν_i is the field of neutrino with the mass m_i and U is the unitary mixing matrix. The field ν_i can be the Dirac field if the total lepton number L is conserved. If there are no conserved lepton numbers, ν_i is the field of the Majorana neutrino, which satisfies the condition

$$\nu_i = \nu_i^c = C \bar{\nu}_i^T,$$

where C is the matrix of the charge conjugation.

Neutrino oscillations is a new phenomenon. There are many discussions of the quantum mechanical problems connected with it (see, for example, [38] and references therein). We will present here the field-theoretical point of view (see [28, 56, 40]), which allow us to obtain the standard formulas for transition probabilities and to understand the origin of oscillations (neutrino, neutral kaons etc).

Flavor neutrinos ν_e , ν_μ and ν_τ are produced in decays of pions, kaons, neutron-rich nuclei in a reactor, neutrino reactions in the sun etc. Let us consider the emission of neutrinos with momentum \vec{p} in a decay

$$a \rightarrow b + l^+ + \nu_l. \quad (30)$$

In the case of the neutrino mixing (29) the state of the final particles is given by

$$|f\rangle = \sum_{i=1}^3 U_{li}^* |\nu_i\rangle |l^+\rangle |b\rangle \langle \nu_i l^+ b | S | a \rangle. \quad (31)$$

Here $\langle \nu_i l^+ b | S | a \rangle$ is the transition matrix element and $|\nu_i\rangle$ is the state of neutrino with the momentum \vec{p} and the energy $E_i = \sqrt{p^2 + m_i^2}$.

From data of the Mainz [41] and the Troitsk [42] tritium experiments the following upper bound on the absolute value of neutrino mass was obtained

$$m_i \leq 2.2 \text{ eV}$$

A more strict bound

$$m_i \leq 0.6 \text{ eV}$$

was found from the analysis of the latest cosmological data [43].

Energies E of neutrinos in neutrino experiments are in the MeV range (solar and reactor experiments) in GeV range (atmospheric, accelerator long baseline experiments) etc. Thus, $(m_i/E)^2 \ll 1$ and we can safely neglect tiny effects of neutrino masses in production processes. For the decay (30) we have

$$\langle \nu_i l^+ b | S | a \rangle \simeq \langle \nu_i l^+ b | S | a \rangle_{m_i^2=0} = \langle \nu_l l^+ b | S | a \rangle_{SM}, \quad (32)$$

where $\langle \nu_l l^+ b | S | a \rangle_{SM}$ is the SM matrix element of the process (30). For the state $|f\rangle$ we have

$$|f\rangle \simeq |\nu_l\rangle |l^+\rangle |b\rangle \langle \nu_l l^+ b | S | a \rangle_{SM}, \quad (33)$$

where

$$|\nu_l\rangle = \sum_{i=1}^3 U_{li}^* |\nu_i\rangle \quad (34)$$

is the state of the flavor neutrino ν_l .¹⁰

The relation (34) is similar to the relations

$$\begin{aligned} |K^0\rangle &= \frac{1}{2N} (|K_S^0\rangle + |K_L^0\rangle) \\ |\overline{K}^0\rangle &= \frac{1}{2N} \frac{p}{q} (|K_S^0\rangle - |K_L^0\rangle), \end{aligned} \quad (35)$$

which connect states of the neutral kaons K^0 and \overline{K}^0 , particles with definite strangeness, with the states of K_S^0 and K_L^0 , particles with definite masses and widths. In (35) $N = \frac{1}{\sqrt{1+|\frac{q}{p}|^2}}$ is the normalizing factor and p/q is a complex parameter.

The relation (35) is based on the Weisskopf-Wigner approximation, which is valid in the case of neutral kaons because the mass difference $\Delta m = m_{K_L} - m_{K_S}$ is much smaller than the kaon mass.

The states of flavor neutrinos (34) are *coherent superpositions* of the states of the neutrino ν_i with definite mass. For the flavor antineutrino $\bar{\nu}_l$ we have

$$|\bar{\nu}_l\rangle = \sum_{i=1}^3 U_{li} |\bar{\nu}_i\rangle, \quad (36)$$

where $|\bar{\nu}_i\rangle$ is the state of antineutrino (in the Dirac case) or neutrino (in the Majorana case) with mass m_i , momentum $p = (\vec{p}, E_i)$ and positive helicity ($|\nu_i\rangle$ in Eq. (34) is the state with negative helicity).

We will consider now the evolution of the flavor states (34) in vacuum. If at $t = 0$ flavor neutrino ν_l is produced, for the neutrino state at a time t we will have

$$|\nu_l\rangle_t = e^{-iH_0 t} |\nu_l\rangle = \sum_{i=1}^2 U_{li}^* e^{-iE_i t} |\nu_i\rangle, \quad (37)$$

¹⁰In the experiments on the direct measurement of the neutrino mass [41, 42] the high energy part of the spectrum of electrons from the decay of tritium is studied. This part of the spectrum corresponds to the emission of neutrinos with small energies. Effect of the distortion of the electron spectrum can be observed if neutrino energy and neutrino mass are of the same order. The relation (32) obviously is not valid in this case.

where H_0 is the free Hamiltonian. Developing E_i over m_i^2 we have

$$E_i \simeq E + \frac{m_i^2}{2E}, \quad (38)$$

where $E = p$ is the energy of the neutrino in the approximation $m_i^2 \rightarrow 0$. From (37) and (38) for the neutrino state at the time t we have

$$|\nu_l\rangle_t = e^{-iEt} \sum_{i=1}^3 e^{-i\frac{m_i^2 t}{2E}} U_{li}^* |\nu_i\rangle. \quad (39)$$

Thus, at a time t *different massive neutrino states* $|\nu_i\rangle$ *acquire different phases*. The phase differences at macroscopic distances $L \simeq t$ can be large. This means that the flavor content of the state $|\nu_l\rangle_t$ can be quite different from the initial state $|\nu_l\rangle$. The differences of the phases of the different neutrino mass components at macroscopic distances is *the physical origin of the phenomenon of neutrino oscillations*.

Let us discuss now a detection process. Neutrinos are detected via the observation of CC and NC weak processes. Taking into account the unitarity of the mixing matrix, from (34) we have

$$|\nu_i\rangle = \sum_l U_{li} |\nu_l\rangle. \quad (40)$$

From (39) and (40) we find

$$|\nu_l\rangle_t = e^{-iEt} \sum_{l'} A(\nu_l \rightarrow \nu_{l'}) |\nu_{l'}\rangle, \quad (41)$$

where

$$A(\nu_l \rightarrow \nu_{l'}) = \sum_{i=1}^3 U_{l'i} e^{-i\frac{m_i^2 t}{2E}} U_{li}^* \quad (42)$$

is the amplitude of the probability to find a state $|\nu_{l'}\rangle$ in the state $|\nu_l\rangle_t$.

Let us consider the CC process

$$\nu_{l'} + N \rightarrow l' + X. \quad (43)$$

If the neutrino state is $|\nu_l\rangle$, the matrix element of the production of the lepton l' in the process (43) is given by

$$\langle l'X|S|\nu_l N\rangle = \sum_i \langle l'X|S|\nu_i N\rangle U_{li}^* U_{li}. \quad (44)$$

Now, taking into account that $(m_i/E)^2 \ll 1$, we have

$$\langle l' X | S | \nu_i N \rangle \simeq \langle l' X | S | \nu_i N \rangle_{m_i^2=0} = \langle l' X | S | \nu_{l'} N \rangle_{SM}, \quad (45)$$

where $\langle l' X | S | \nu_{l'} N \rangle_{SM}$ is the SM matrix element of the process (43).

Using now the unitarity of the mixing matrix, from (44) and (45) we obtain

$$\langle l' X | S | \nu_l N \rangle \simeq \delta_{ll'} \langle l' X | S | \nu_{l'} N \rangle_{SM}. \quad (46)$$

From (41), (42) and (46) for the probability of the transition of the flavor neutrino ν_l , produced in a standard CC weak process, to the flavor neutrino $\nu_{l'}$, detected in a standard CC weak process, we obtain the following expression

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_{i=1}^3 U_{l'i} e^{-i\Delta m_{i1}^2 \frac{L}{2E}} U_{li}^* \right|^2, \quad (47)$$

where $\Delta m_{i1}^2 = m_i^2 - m_1^2$ ¹¹ L is the distance between neutrino source and neutrino detector ($L \simeq t$, where t is the time between production and detection of neutrinos¹²)

Taking into account the unitarity of the mixing matrix we can present the transition probability in the form

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \delta_{ll'} + \sum_{i=2,3} U_{l'i} (e^{-i\Delta m_{i1}^2 \frac{L}{2E}} - 1) U_{li}^* \right|^2. \quad (48)$$

The following remarks are in order.

1. For the probability of the antineutrino transition $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ we have

$$P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) = \left| \delta_{ll'} + \sum_{i=2,3} U_{l'i}^* (e^{-i\Delta m_{i1}^2 \frac{L}{2E}} - 1) U_{li} \right|^2 \quad (49)$$

2. From the comparison of Eq.(48) and Eq.(49) we conclude that the following relation

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l) \quad (50)$$

holds. The relation (50) is a consequence of the CPT theorem.

¹¹We label neutrino masses in such a way that $m_1 < m_2 < m_3$

¹²The relation $L = t$ was used in the long baseline accelerator K2K experiment[44] in order to provide timing information for the selection of neutrino events in the Super-Kamiokande detector.

3. In the case of the CP invariance in the lepton sector the mixing matrix U is real in the Dirac case. In the Majorana case the mixing matrix satisfies the condition

$$U_{\alpha i} = U_{\alpha i}^* \eta_i, \quad (51)$$

where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino ν_i . From (48), (49) and (51) it follows that in the case of the CP invariance in the lepton sector the probabilities of the transitions $\nu_l \rightarrow \nu_{l'}$ and $\bar{\nu}_l \rightarrow \bar{\nu}_{l'}$ are equal:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \quad (52)$$

4. If the number of the neutrinos ν_i is larger than three, for the mixing we have

$$\nu_{lL} = \sum_{i=1}^{3+n_s} U_{li} \nu_{iL}; \quad \nu_{sL} = \sum_{i=1}^{3+n_s} U_{si} \nu_{iL}, \quad (53)$$

where U is the unitary $(3 + n_s) \times (3 + n_s)$ mixing matrix and ν_{sL} ($s = s_1, \dots, s_{n_s}$) is a sterile field (the field which does not enter into CC and NC Lagrangians). In the case of small masses m_i the states of the sterile neutrinos are *defined* as follows

$$|\nu_s\rangle = \sum_{i=1}^{3+n_s} U_{si}^* |\nu_i\rangle \quad (54)$$

From the unitarity of the mixing matrix for the state of neutrino with definite mass we have in this case

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle, \quad (55)$$

where index α takes the values e, μ, τ, s_1, \dots

The transition probability in the general case of the transitions into active (flavor) and sterile states is given by the expression

$$P(\nu_{\alpha} \rightarrow \nu_{\alpha'}) = |\delta_{\alpha'\alpha} + \sum_{i=2}^{3+n_s} U_{\alpha'i} (e^{-i\Delta m_{i1}^2 \frac{L}{2E}} - 1) U_{\alpha i}^*|^2. \quad (56)$$

5. If neutrinos are detected via the observation of NC processes, the observed number of the NC events is the product of the total probability of the transition of the initial neutrino ν_l into all flavor states $\sum_{l'=e,\mu,\tau} P(\nu_l \rightarrow \nu_{l'})$ and the number of the events expected in case of no neutrino oscillations. If there are no transitions into sterile states, from the conservation of probability we have

$$\sum_{l'=e,\mu,\tau} P(\nu_l \rightarrow \nu_{l'}) = 1.$$

Thus, there are no neutrino oscillations in this case.

If there are transitions into sterile states, we have

$$\sum_{l'=e,\mu,\tau} P(\nu_l \rightarrow \nu_{l'}) = 1 - \sum_s P(\nu_l \rightarrow \nu_s),$$

where $\sum_s P(\nu_l \rightarrow \nu_s)$ is the total probability of the transition of the flavor neutrino ν_l into all possible sterile states.

6. Neutrino mass term does not conserve flavor lepton numbers L_e , L_μ , and L_τ . What are flavor neutrinos ν_e , ν_μ and ν_τ ? It is clear from the previous discussion that flavor neutrino ν_l is a particle which is emitted together l^+ in CC production processes, produces l^- in CC detection processes etc. Neutrino oscillations take place because *the states of flavor neutrinos are not states with definite masses*.
7. In the simplest case of the transition between two flavor neutrinos index i in (48) and (49) takes the value 2. For $l' \neq l$ we have

$$P(\nu_l \rightarrow \nu_{l'}) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos \Delta m^2 \frac{L}{2E}\right); \quad (l' \neq l). \quad (57)$$

Here $\Delta m^2 = m_2^2 - m_1^2$ and θ is the mixing angle ($|U_{l'2}|^2 = \sin^2 \theta$, $|U_{l2}|^2 = \cos^2 \theta$).

It is easy to see that in the two-neutrino case the following relations are automatically satisfied

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}). \quad (58)$$

Thus, the CP violation in the lepton sector can not be revealed if there are transitions only between two neutrinos.

For the ν_l ($\nu_{l'}$) survival probability from the conservation of probability we find

$$P(\nu_l \rightarrow \nu_l) = 1 - P(\nu_l \rightarrow \nu_{l'}); \quad P(\nu_{l'} \rightarrow \nu_{l'}) = 1 - P(\nu_{l'} \rightarrow \nu_l) \quad (59)$$

Thus, we have

$$P(\nu_l \rightarrow \nu_l) = P(\nu_{l'} \rightarrow \nu_{l'}) \quad (60)$$

Let us stress that this relation is valid only in the case of the transitions between two types of neutrinos.

The formulas (57) and (59) describe periodical transitions $\nu_l \rightleftharpoons \nu_{l'}$ with the oscillation length (in the units $\hbar = c = 1$) given by the expression

$$L_0 = 4\pi \frac{E}{\Delta m^2} \quad (61)$$

The oscillation length can be also presented in the form

$$L_0 = 2.48 \frac{E}{\Delta m^2} m, \quad (62)$$

where E is neutrino energy in MeV and Δm^2 is neutrino mass-squared difference in eV^2 .

From (62) it follows that in order to observe neutrino oscillations the condition

$$L \gtrsim L_0; \quad \text{or} \quad \Delta m^2 \gtrsim \frac{E}{L}$$

must be satisfied.

4 Evidence for neutrino oscillations

In the solar neutrino experiments Homestake [4], GALLEX-GNO [5], SAGE [6] and Super-Kamiokande [7], neutrinos are detected in different energy ranges. In all these experiments the observed event rates are significantly smaller than the rates predicted by the Standard Solar Model [45]. This suppression can be naturally explained by the transition of the solar ν_e 's into other neutrino neutrinos.

Impressive evidence of such transition was obtained recently in the SNO experiment [2]. In this experiment high energy solar neutrinos, produced in the decay ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$, were detected via the observation of the CC process

$$\nu_e + d \rightarrow e^- + p + p \quad (63)$$

and NC process

$$\nu_l + d \rightarrow \nu_l + n + p \quad (l = e, \mu, \tau) \quad (64)$$

The measurement of the CC event rate allows to determine the total flux of the solar ν_e 's on the earth. In the SNO experiment it was found

$$\Phi_{\nu_e}^{\text{SNO}} = (1.59_{-0.07}^{+0.09}(\text{stat.})_{-0.08}^{+0.06}(\text{syst.})) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}. \quad (65)$$

The measurement of the NC event rate allows to determine *the total flux of all high energy flavor neutrinos ν_e , ν_μ and ν_τ on the earth.* In the SNO experiment it was obtained

$$\sum_{l=e,\mu,\tau} \Phi_{\nu_l}^{\text{SNO}} = (5.21 \pm 0.27 \pm 0.38) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}. \quad (66)$$

The SNO results clearly demonstrate that original solar ν_e 's on the way from the production region in the central zone of the sun to the earth are transferred into ν_μ and ν_τ .

The data of all solar neutrino experiments can be described by the two-neutrino MSW [46] transitions in matter. From the analysis of the solar neutrino data in the preferable LMA region the following best-fit values of the two neutrino oscillation parameters were found [2]:

$$\Delta m_{\text{sol}}^2 = 5 \cdot 10^{-5} \text{eV}^2; \tan^2 \theta_{\text{sol}} = 0.34; (\chi_{\text{min}}^2 = 57/72 \text{ d.o.f.}). \quad (67)$$

Further compelling evidence in favor of neutrino oscillations was obtained recently in the reactor KamLAND experiment [3]. For the value of Δm_{sol}^2 , given by (67), the oscillation length is in the range $L_0 \simeq (100 - 200) \text{ km}$. In the KamLAND experiment $\bar{\nu}_e$'s from many reactors in Japan and Korea are detected via the observation of e^+ and n , produced in the process

$$\bar{\nu}_e + p \rightarrow e^+ + n.$$

The threshold of this process is equal to 1.8 MeV. About 80% of the total number of the events is due to antineutrinos from 26 reactors within the

distances 138-214 km. For the ratio of total numbers of the observed and expected events the following value was found

$$\frac{N_{obs}}{N_{exp}} = 0.611 \pm 0.085 \pm 0.041. \quad (68)$$

From the analysis of all solar and KamLAND data the following best-fit values of neutrino oscillation parameters were found [2]:

$$\Delta m_{sol}^2 = 7.1 \cdot 10^{-5} \text{eV}^2; \tan^2 \theta_{sol} = 0.41; \quad (69)$$

Another important evidence in favor of neutrino oscillations was obtained in the atmospheric neutrino experiments [1, 8, 9]. Atmospheric neutrinos are produced mainly in the decays of pions (produced in the interaction of the cosmic rays with nuclei in the earth atmosphere) and subsequent decays of muons: $\pi \rightarrow \mu \nu_\mu$; $\mu \rightarrow e \nu_\mu \nu_e$. In the Super-Kamiokande (S-K) experiment [1] significant dependence on the azimuthal angle θ_z of the Multi-Gev muon events ($E_{vis} \geq 1.3 \text{ GeV}$) was observed. Neutrino coming to the detector from above ($0.2 \leq \cos \theta_z \leq 1$) travel the distances L in the range $20 \text{ km} \lesssim L \lesssim 500 \text{ km}$ and neutrinos which enter into the detector from below ($-1 \leq \cos \theta_z \leq -0.2$) travel the distances in the range $500 \text{ km} \lesssim L \lesssim 13000 \text{ km}$. For the ratio of the total number of the up-going ν_μ to the total number of the down-going ν_μ in the Super-Kamiokande experiment the following value

$$\left(\frac{U}{D} \right)_\mu = 0.54 \pm 0.04 \pm 0.01. \quad (70)$$

was found.

The observation of the large up-down asymmetry clearly demonstrates the dependence of the number of the muon neutrinos on the distance they travel from the production region in the atmosphere to the detector.

The S-K data [1] and data of other atmospheric neutrino experiments (SOUDAN 2 [8], MACRO [9]) are perfectly described, if we assume two-neutrino $\nu_\mu \rightleftharpoons \nu_\tau$ oscillations. From the analysis of the S-K data the following best-fit values of the neutrino oscillation parameters

$$\Delta m_{atm}^2 = 2 \cdot 10^{-3} \text{eV}^2; \sin^2 2\theta_{atm} = 1.0 \quad (\chi_{min}^2 = 170.8/170 \text{ d.o.f.}) \quad (71)$$

were found [1].

Atmospheric neutrino evidence in favor of neutrino oscillations have been confirmed by the accelerator long baseline K2K experiment[44]. In this experiment muon neutrinos with average energy 1.3 GeV, produced at the KEK facility, were detected by the Super-Kamiokande detector at the distance of about 250 km. The total number of the observed muon events was equal to 56. The expected number of the muon events was equal to $80.1^{+6.2}_{-5.4}$.

Thus, in the K2K experiment indications in favor of the disappearance of the accelerator ν_μ were obtained. From the two-neutrino analysis of the data the following best-fit values of the oscillation parameters were found

$$\sin^2 2\theta_{K2K} = 1 : \Delta m_{K2K}^2 = 2.8 \cdot 10^{-3} \text{ eV}^2. \quad (72)$$

These values are compatible with the values of the oscillation parameters that were found from the analysis of the S-K atmospheric neutrino data.

5 Neutrino oscillations in the framework of three-neutrino mixing

We will consider here neutrino oscillations in the framework of the three-neutrino mixing

$$\nu_{iL} = \sum_{i=1}^3 U_{li} \nu_{iL}, \quad (73)$$

where U is 3×3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [34, 37, 47] mixing matrix.

From the analysis of the data of neutrino oscillation experiments it was established that two independent neutrino mass-squared differences Δm_{sol}^2 and Δm_{atm}^2 satisfy the hierarchy

$$\Delta m_{sol}^2 \ll \Delta m_{atm}^2. \quad (74)$$

In the framework of the three-neutrino mixing neutrino oscillation data are compatible with two types of neutrino mass spectra:

I. “Normal” mass spectrum

$$\Delta m_{21}^2 \simeq \Delta m_{sol}^2; \Delta m_{32}^2 \simeq \Delta m_{atm}^2; \quad (75)$$

II. “Inverted” mass spectrum ¹³

$$\Delta m_{32}^2 \simeq \Delta m_{\text{sol}}^2; \Delta m_{21}^2 \simeq \Delta m_{\text{atm}}^2; \quad (76)$$

Let us consider neutrino oscillations in the atmospheric range of L/E ($L/E \simeq 10^3$) under the assumption of the normal neutrino mass spectrum. In this range the “solar” phase $\Delta m_{21}^2 L/2E$ is small and we can neglect the $i = 2$ term in Eq. (48) and Eq. (49). For the $\bar{\nu}_e$ survival probability we have

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} B_{ee} (1 - \cos \Delta m_{32}^2 \frac{L}{2E}), \quad (77)$$

where

$$B_{ee} = 4 |U_{e3}|^2 (1 - |U_{e3}|^2). \quad (78)$$

is the amplitude of oscillations.

In two reactor experiments CHOOZ [48] and Palo Verde [49] the search for neutrino oscillations in the atmospheric range of L/E have been performed. The reactor-detector distances in these experiments were about 1 km. No disappearance of reactor $\bar{\nu}_e$ ’s was found. From the exclusion plot, obtained from the analysis of the data of the CHOOZ experiment, at the point $\Delta m_{32}^2 = 2 \cdot 10^{-3} \text{eV}^2$ (the S-K best-fit value) for the amplitude B_{ee} we have the following upper bound

$$B_{ee} \lesssim 2 \cdot 10^{-1} \quad (79)$$

From (78) and (79) it follows that the parameter $|U_{e3}|^2$ can be small or large (close to 1). Taking into account the solar neutrino data we can exclude the large values of $|U_{e3}|^2$. Thus, we have

$$|U_{e3}|^2 = \sin^2 \theta_{13} \lesssim 5 \cdot 10^{-2}, \quad (80)$$

where θ_{13} is the 1-3 mixing angle.

Neglecting the solar term in (48) and (49), for the probability of $\nu_\mu \rightarrow \nu_\tau$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$) transition in the atmospheric range we find the following expression

¹³Notice that for inverted spectrum neutrino masses are often labeled in such a way that $m_3 < m_1 < m_2$. In this case

$$\Delta m_{21}^2 \simeq \Delta m_{\text{sol}}^2; |\Delta m_{31}^2| \simeq \Delta m_{\text{atm}}^2$$

$$P(\nu_\mu \rightarrow \nu_\tau) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) = \frac{1}{2} A_{\tau\mu} \left(1 - \cos \Delta m_{32}^2 \frac{L}{2E} \right), \quad (81)$$

where the oscillation amplitude is given by the expression

$$A_{\tau\mu} = 4 |U_{\tau 3}|^2 |U_{\mu 3}|^2. \quad (82)$$

In the standard parametrization of the neutrino mixing matrix we have

$$U_{\mu 3} = \sqrt{1 - |U_{e3}|^2} \sin \theta_{23}; \quad U_{\tau 3} = \sqrt{1 - |U_{e3}|^2} \cos \theta_{23}, \quad (83)$$

where θ_{23} is the 2-3 mixing angle. Thus, the amplitude of the $\nu_\mu \rightleftharpoons \nu_\tau$ oscillations is given by

$$A_{\tau;\mu} = (1 - |U_{e3}|^2)^2 \sin^2 2\theta_{23} \quad (84)$$

In the expression for the $\bar{\nu}_e$ survival probability in vacuum in the “KamLAND range” of L/E the effect of large neutrino mass-squared difference Δm_{32}^2 is averaged out. We have

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 P^{(1,2)}(\bar{\nu}_e \rightarrow \bar{\nu}_e), \quad (85)$$

where

$$P^{(1,2)}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{12} \left(1 - \cos \Delta m_{21}^2 \frac{L}{2E} \right), \quad (86)$$

is the two-neutrino transition probability in vacuum and θ_{12} is 1-2 mixing angle.

The ν_e survival probability in matter is given by the expression [50]

$$P_{\text{mat}}(\nu_e \rightarrow \nu_e) = |U_{e3}|^4 + (1 - |U_{e3}|^2)^2 P_{\text{mat}}^{(1,2)}(\nu_e \rightarrow \nu_e), \quad (87)$$

where $P_{\text{mat}}^{(1,2)}(\nu_e \rightarrow \nu_e)$ is the two-neutrino survival probability in matter, which depend on parameters Δm_{21}^2 and $\tan^2 \theta_{12}$.

From the data of the CHOOZ experiment it follows that the parameter $|U_{e3}|^2$ is small (see (80)). If we will neglect the contribution of $|U_{e3}|^2$ to the transition probabilities, the following simple picture of neutrino oscillations emerges (see [51]):

1. In the atmospheric range of L/E there are no transitions $\nu_e \rightarrow \nu_{\mu,\tau}$ and neutrino oscillations in this range are pure *two-neutrino* $\nu_\mu \rightleftharpoons \nu_\tau$ oscillations. Taking into account (81) and (84), we have

$$\sin^2 2\theta_{23} \simeq \sin^2 2\theta_{\text{atm}}; \quad \Delta m_{32}^2 \simeq \Delta m_{\text{atm}}^2. \quad (88)$$

2. The solar ν_e are transferred inside of the sun into ν_μ and ν_τ . These transitions are described by the *two-neutrino survival probability in matter*. We have

$$\tan^2 \theta_{12} \simeq \tan^2 \theta_{\text{sol}}; \quad \Delta m_{12}^2 \simeq \Delta m_{\text{sol}}^2. \quad (89)$$

3. In the KamLAND range of L/E $\nu_e \rightleftharpoons \nu_{\mu,\tau}$ oscillations take place. The $\bar{\nu}_e$ survival probability is given in this range by *the standard two-neutrino expression*. Thus we have

$$\sin^2 2\theta_{12} \simeq \sin^2 2\theta_{K-L}; \quad \Delta m_{21}^2 \simeq \Delta m_{K-L}^2. \quad (90)$$

4. In the standard parametrization

$$U_{e3} = \sin \theta_{13} e^{-i\delta}, \quad (91)$$

where δ is the CP phase. Thus, in the leading approximation the effects of the CP violation in the lepton sector can not be revealed.

This picture is in a good agreement with the existing neutrino oscillation data. The next important step in the investigation of neutrino oscillations will be the search for possible small effects beyond the leading approximation.

First of all it is necessary to measure the value of the parameter $\sin^2 \theta_{13}$ (or to improve the bound (80)). Several groups are considering a possibility to perform a new reactor experiment of the CHOOZ type with a reactor-detector distance about 1 km (see Reactor White Paper [52]). Information about the parameter $\sin^2 \theta_{13}$ can be also obtained from experiments on the search for $\nu_\mu \rightarrow \nu_e$ oscillations in the future long baseline accelerator neutrino experiments. Such experiments will be done by the MINOS collaboration [53] and under preparation at the JPARC facility [54].

6 Neutrinoless double β -decay

The search for neutrinoless double β -decay ($0\nu\beta\beta$ -decay)

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-. \quad (92)$$

is one of the most important problem of today's neutrino physics. The observation of this decay would be a proof that the total lepton number is not

conserved and massive neutrinos ν_i are Majorana particles. The half-life of the process is given by (see [55, 56, 57]):

$$\frac{1}{T_{1/2}^{0\nu}(A, Z)} = |m_{ee}|^2 |M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z). \quad (93)$$

Here

$$m_{ee} = \sum_i U_{ei}^2 m_i \quad (94)$$

is the effective Majorana mass, $|M^{0\nu}(A, Z)|$ is the nuclear matrix element and $G^{0\nu}(E_0, Z)$ is known phase-space factor (E_0 is the energy release). Many experiments on the search for $0\nu\beta\beta$ decay are going on at present (see [60]). The best lower bound on the half-life was reached in the ^{76}Ge Heidelberg-Moscow experiment [61]:

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 1.9 \cdot 10^{25} \text{ y} \quad (90\% \text{ CL}) \quad (95)$$

Taking into account the results of different calculations of the nuclear matrix element, from this bound for the effective Majorana mass the following upper bound can be inferred

$$|m_{ee}| \lesssim (0.3 - 1.2) \text{ eV}. \quad (96)$$

Several new experiments on the search for the neutrinoless double β -decay of ^{76}Ge , ^{130}Te , ^{136}Xe , ^{100}Mo and other nuclei are in preparation (see [60]). In these experiments significant improvement in the sensitivity to $|m_{ee}|$ (one-two orders of the magnitude) is expected.

The effective Majorana mass $|m_{ee}|$ are determined by the absolute value of neutrino masses and elements U_{ei}^2 . For the neutrino masses we have

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}; \quad m_3 = \sqrt{m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2}, \quad (97)$$

where parameters Δm_{21}^2 and Δm_{32}^2 can be determined from the data of neutrino oscillation experiments. The elements U_{e1} and U_{e2} are given by

$$U_{e1} = \sqrt{1 - |U_{e3}|^2} \cos \theta_{12} e^{i\alpha_1} \quad U_{e2} = \sqrt{1 - |U_{e3}|^2} \sin \theta_{12} e^{i\alpha_1}, \quad (98)$$

where α_1 and α_2 are Majorana CP phases. The value of the parameter $\sin^2 \theta_{12}$ and upper bound of the parameter $|U_{e3}|^2$ can be determined from the neutrino oscillation data.

The possible value of the effective Majorana mass strongly depends on the minimal neutrino mass and character of neutrino mass spectrum (see [58, 59]). In the case of neutrino mass hierarchy

$$m_1 \ll m_2 \ll m_3. \quad (99)$$

neutrino masses m_2 and m_3 are given by

$$m_2 \simeq \sqrt{\Delta m_{\text{sol}}^2}; \quad m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2} \quad (100)$$

and minimal neutrino mass is small ($m_1 \ll \sqrt{\Delta m_{\text{sol}}^2}$). For the effective Majorana mass we have in this case the bound

$$|m_{ee}| \lesssim 4.6 \cdot 10^{-3} \text{ eV}, \quad (101)$$

which is much smaller than the sensitivity of the future experiments on the search for $0\nu\beta\beta$ -decay.

In the case of the inverted mass hierarchy

$$m_1 \ll m_2 < m_3. \quad (102)$$

for neutrino masses we have

$$m_2 \simeq m_3 \simeq \sqrt{\Delta m_{\text{atm}}^2} \quad (103)$$

and $m_1 \ll \sqrt{\Delta m_{\text{atm}}^2}$.

The effective Majorana mass $|m_{ee}|$ in this case is given by

$$|m_{ee}| \simeq \sqrt{\Delta m_{\text{atm}}^2} (1 - \sin^2 2\theta_{\text{sol}} \sin^2 \alpha)^{\frac{1}{2}}, \quad (104)$$

where $\alpha = \alpha_2 - \alpha_1$ is Majorana CP-phase difference. From (104) we obtain the range

$$2.9 \cdot 10^{-2} \text{ eV} \lesssim |m_{ee}| \lesssim 5.5 \cdot 10^{-2} \text{ eV}. \quad (105)$$

The value of the effective Majorana mass in this range can be reached in the future $0\nu\beta\beta$ -decay experiments.

If the minimal neutrino mass is relatively large ($m_1 \gg \sqrt{\Delta m_{\text{atm}}^2}$) in this case neutrino masses are practically degenerate. For the effective Majorana mass we have in this case

$$0.65 m_1 \leq |m_{ee}| \leq m_1 \quad (106)$$

Let us notice that in the future tritium experiment KATRIN [62] the sensitivity $m_1 \simeq 0.25 \text{ eV}$ is planned to be reached.

7 A remark on the see-saw mechanism

The understanding of the physical origin of neutrino masses and mixing is the main purpose of modern experimental and theoretical neutrino physics. From existing data we know that neutrino masses are much smaller than masses of quarks and leptons. For example, for the masses of the fundamental fermions of the third family we have

$$\begin{aligned} m_t &= 174.3 \pm 5.1 \text{ GeV}; \quad m_b = (4.0 - 4.5) \text{ GeV}; \\ m_\tau &= 1776.99 \pm 0.29 \text{ MeV}; \quad m_3 \leq 2.2 \text{ eV} \quad (0.6 \text{ eV}) \end{aligned} \quad (107)$$

The original Standard Model [29, 30, 31] was build for the case of massless two-component neutrinos. In the framework of the SM there is, however, no principle which force neutrino masses to be equal to zero (like gauge invariance for the photon). In the SM masses of quarks and leptons are parameters, generated by the Higgs mechanism with one Higgs doublet. On the same footing masses of neutrinos can be generated. We may expect, however, that neutrino masses, generated by the standard Higgs mechanism, are of the same order of magnitude as masses of other family partners (quarks and lepton). Thus, to be in agreement with experimental data we need in this case an additional beyond the SM mechanism which provide smallness of neutrino masses.

One of the most attractive mechanism of such type is the see-saw [63]. Let us consider the simplest case of one generation. The Dirac mass term

$$\mathcal{L}^D = -m \bar{\nu}_R \nu_L + \text{h.c.} \quad (108)$$

with m of the order of a lepton or quark mass can be generated by the standard Higgs mechanism.

Let us assume that there exists a new mechanism which does not conserve lepton number and generate the right-handed Majorana mass term ¹⁴

$$\mathcal{L}_R^M = -\frac{1}{2} M \bar{\nu}_R (\nu_R)^c + \text{h.c.}, \quad (109)$$

where ν_R is $SU(2)$ singlet, $(\nu_R)^c = C \bar{\nu}_R^T$ is the charge conjugated field and $M \gg m$ (usually it is assumed that $M \simeq M_{\text{GUT}} \simeq (10^{15} - 10^{16}) \text{ GeV}$).

¹⁴The conservation of the electric charge does not allow such a term for quarks and charged leptons

After the diagonalization of the total mass term we have

$$\begin{aligned}\nu_L &= i \cos \theta \nu_{1L} + \sin \theta \nu_{2L} \\ (\nu_R)^c &= -i \sin \theta \nu_{1L} + \cos \theta \nu_{2L},\end{aligned}\tag{110}$$

where ν_1 is the field of the Majorana neutrino with the mass

$$m_1 \simeq \frac{m^2}{M} \ll m\tag{111}$$

and ν_2 is the field of a heavy neutral Majorana lepton with the mass

$$m_2 \simeq M \gg m\tag{112}$$

The mixing angle θ is given by the relation

$$\tan 2\theta = \frac{2m}{M} \ll 1\tag{113}$$

In the general case of three families the see-saw mechanism generates three light Majorana masses and three heavy Majorana masses. Let us stress that if neutrino masses are of the standard see-saw origin :

- Neutrinos with definite masses are Majorana particles.
- There are three light neutrinos.
- Three heavy Majorana leptons, see-saw partners of neutrinos, must exist.

The existence of the heavy Majorana particles, could be a source of the baryon asymmetry of the Universe (see [64]).

8 Conclusion

The history of neutrino demonstrates the complicated and unpredictable way of science. We may expect further surprises from neutrinos. The discovery of neutrino oscillations, driven by small neutrino masses and neutrino mixing, requires apparently either additional to SM mechanism of neutrino mass generation or a completely new mechanism, very different from the mechanism of generation of masses of quarks and leptons. In order to reveal the true nature of the new phenomenon we definitely need additional information.

The great progress in neutrino physics, reached in the recent years, dictates the nearest problems to be solved:

- What is the nature of neutrinos with definite masses ?

Are they Dirac particles, possessing the conserved lepton number, or truly neutral Majorana particles? Investigation of neutrino oscillations do not allow to answer this question [65]. The answer to this fundamental question can be obtained in experiments on the search for neutrinoless double β -decay of some even-even nuclei.

- What is the value of the minimal neutrino mass ?

Neutrino oscillation experiments allow to determine only neutrino mass-squared differences. Information on the minimal neutrino mass can be obtained from the future tritium KATRIN experiment and from cosmological data .

- What is the value of the parameter $\sin \theta_{13}$?

- What is the value of the leptonic CP phase?

The answer to this question apparently can be obtained in future experiments at the JPARC facility [54], in the off-axes neutrino experiments [66] in the β -beam neutrino experiments (see [67]) and in the Neutrino Factory experiments (see [68]).

- Is the number of massive neutrino equal to the number of flavor neutrinos (three)?

Indication in favor of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations with the third “large” neutrino mass-squared difference ($\Delta m^2 \simeq 1\text{eV}^2$) was obtained in the LSND experiment [69]. If this indication will be confirmed, we need (at least) four massive neutrinos to describe experimental data. The situation apparently will be clarified by the MiniBooNE experiment at Fermilab [70].

It is my pleasure to thank W. Alberico, M. Fabbrichesi and S.Petcov for useful discussions. I acknowledge the support of the program “Rientro dei cervelli”.

References

- [1] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); S. Fukuda *et al.*, Phys. Rev. Lett. **82**, 2644 (1999); S. Fukuda *et al.*, Phys. Rev. Lett. **85**, 3999-4003 (2000).
- [2] SNO collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **87**, 071301 (2001) Q.R. Ahmad *et al.*, Phys.Rev.Lett. **89**, 011301 (2002); nucl-ex/0204008. Q.R. Ahmad *et al.*, Phys.Rev.Lett **89**, 011302 (2002); nucl-ex/0204009. Phys.Rev.Lett to be published, nucl-ex/0309004.
- [3] KamLAND collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003) hep-ex/0212021.
- [4] B. T. Cleveland *et al.*, Astrophys. J. **496** (1998) 505.
- [5] GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. **B 447** (1999) 127 ; GNO Collaboration, M. Altmann *et al.*, Phys. Lett. **B 490** (2000) 16 ; Nucl.Phys.Proc.Suppl. **91** (2001) 44.
- [6] SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. **C 60** (1999) 055801 ; Nucl.Phys.Proc.Suppl. **110** (2002) 315;
- [7] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86** (2001) 5651; M.Smy, hep-ex/0208004.
- [8] Soudan 2 Collaboration, W.W.M.Allison *et al.*, Physics Letters **B 449** (1999) 137;
- [9] MACRO Collaboration, M.Ambrosio et al. hep-ex/0106049; Phys. Lett. B517 (2001) 59 M. Ambrosio et al. NATO Advanced Research Workshop on Cosmic Radiations, Oujda (Morocco), 21-23 March, 2001.
- [10] P.A.M. Dirac, Proc. Roy. Soc. **A 126** (1930) 369.
- [11] S. Weinberg “The Quantum Theory of Fields” vol. I, Cambridge University Press, 2000.
- [12] P.A.M. Dirac, Proc. Roy. Soc. **A 133** (1931) 60.
- [13] C.D. Anderson, Science **76** (1932) 238.

- [14] . J.Chadwick, Proc. Roy. Soc.,**A**, **136** (1932) 692.
- [15] C.S. Wu *et al.*, Phys. Rev. **105** (1957) 1413.
- [16] T.D. Lee and C.N. Yang, Phys. Rev. **104** (1956) 254.
- [17] L. Landau, Nucl. Phys. **3** (1957) 127 (1957).
- [18] T.D. Lee and C.N. Yang, Phys. Rev. **105** (1957) 1671 .
- [19] A. Salam, Il Nuovo Cim. **5** (1957) 299 .
- [20] W.Pauli, Handbuch der Physik. vol.24, Part I (Springer, Berlin, 1933)
- [21] M. Goldhaber, L. Grodzins and A.W. Sunyar, Phys. Rev. **109** (1958) 1015 .
- [22] R.P. Feynman and M. Gell-Mann, Phys. Rev. **109** (1958) 193 .
- [23] E.C.G. Sudarshan and R. Marshak, Phys. Rev. **109** (1958) 1860 .
- [24] G. Danby *et al.* Phys. Rev. Lett. **9** (1962) 36 .
- [25] B. Pontecorvo, JETP **37** (1959) 1751 .
- [26] N.Cabibbo, Phys. Rev. Lett. **10** (1963) 531.
- [27] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. **D2** (1970) 1285.
- [28] S.M. Bilenky and B.Pontecorvo, Phys. Rep. **41** (1978) 225 .
- [29] S.L. Glashow, Nucl. Phys. **22** (1961) 597 .
- [30] S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264 .
- [31] A. Salam, Proc. of the 8th Nobel Symposium on *Elementary Particle Theory, Relativistic Groups and Analyticity*, edited by N. Svartholm, 1969.
- [32] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.
- [33] B. Pontecorvo, J. Exptl. Theoret. Phys. **33** (1957) 549. [Sov. Phys. JETP **6** (1958) 429].

- [34] B. Pontecorvo, J. Exptl. Theoret. Phys. **34** (1958) 247 [Sov. Phys. JETP **7** (1958) 172].
- [35] F.Reines and C.L. Cowan, Phys. Rev. **92** (1953) 830; Phys. Rev. **113** (1959) 273 .
- [36] R. Davis, Bull. Am. Phys. Soc., Washington meeting (1959)
- [37] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **53** (1967) 1717 [Sov. Phys. JETP **26** (1968) 984].
- [38] C. Giunti, hep- ph/0311241.
- [39] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59** (1987) 671.
- [40] S. M. Bilenky and C. Giunti, Int. J. Mod. Phys. **A16** (2001) 3931, hep-ph/0102320.
- [41] Ch. Weinheimer , Proceedings of the 20th International Conference on Neutrino Physics and Astrophysics, *Neutrino 2002* (Munich, Germany) May 25-30, 2002.
- [42] V. Lobashev *et al.*, Nucl. Phys. Proc. Suppl. **91** (2001) 280.
- [43] M. Tegmark et al., astro-ph/0310723.
- [44] K2K collaboration, M.H.Ahn *et al.*, Phys.Rev.Lett. **90** (2003) 041801; hep-ex/0212007.
- [45] J. N. Bahcall, M. H. Pinsonneault and S. Basu, Astrophys. J. **555** (2001). 990 .
- [46] L. Wolfenstein, Phys. Rev. D **17** (1978) 2369 ; Phys. Rev. D **20** (1979) 2634; S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. **42** (1985) 1441 [Sov. J. Nucl. Phys. **42** (1985) 913]; Il Nuovo Cim. C **9** (1986) 17 ; Zh. Eksp. Teor. Fiz. **91** (1986) 7 [Sov. Phys. JETP **64**(1986) 4].
- [47] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
- [48] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **466** (1999) 415.

- [49] F. Boehm, J. Busenitz et al., Phys. Rev. Lett. **84** (2000) 3764 ; Phys. Rev. D **62** (2000) 072002.
- [50] X. Shi and D.N. Schramm, Phys. Lett. **B 283** (1992) 305.
- [51] S.M. Bilenky and C.Giunti, Phys. Lett. **B444** (1998) 379-386; hep-ph/9802201.
- [52] <http://www.hep.anl.gov/minos/reactor13/white.html>
- [53] MINOS Collaboration, K. Lang *et al.*, Int.J.Mod.Phys. **A18** (2003) 3857.
- [54] T. Kobayashi, Nucl.Phys.Proc.Suppl. 111 (2002) 163
- [55] M. Doi, T. Kotani and E. Takasugi, Progr. Theor. Phys. Suppl. **53** (1985) 1.
- [56] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys.**59** (1987) 671.
- [57] S. R. Elliott and P. Vogel, Annu. Rev. Nucl. Part. Sci. **52** (2002), hep-ph/0202264.
- [58] S.M. Bilenky, C. Giunti, J.A. Grifols and E. Masso, Phys. Rept. **379** (2003) 69-148, hep-ph/0211462.
- [59] S. Pascoli , S.T. Petcov, Proceedings of 10th International Workshop on Neutrino Telescopes, Venice, Italy, 11-14 Mar 2003 (vol. 1p. 301); hep-ph/0308034.
- [60] G. Gratta, Proceedings of the XXI International Symposium on Lepton and Photon Interactions at High Energies, 1-16 August 2003, Fermilab, Batavia, Illinois USA.
- [61] Heidelberg-Moscow collaboration, H. V. Klapdor-Kleingrothaus *et al.*, Eur. Phys. J. **A 12**,(2001) 147.
- [62] KATRIN collaboration, A.Osipowicz *et al.*, hep-ex/0109033.
- [63] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, p. 315, edited by F. van Nieuwenhuizen and D. Freedman, North Holland, Amsterdam, 1979; T. Yanagida, Proc. of the *Workshop on Unified Theory and the*

- Baryon Number of the Universe*, KEK, Japan, 1979; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
- [64] W. Buchmuller, P. Di Bari, M. Plumacher, hep-ph/0401240.
 - [65] S.M. Bilenky, J. Hosek and S.T. Petcov, Phys. Lett.**B94** (1980) 495.
 - [66] G. Feldman , Proceedings of the International Workshop “Neutrino oscillations in Venice”, 3-5 December, 2003.
 - [67] J. Burguet-Castell, D. Casper, J.J. Gomez-Cadenas, P.Hernandez, F.Sanchez, hep-ph/0312068.
 - [68] M. Lindner Int.J.Mod.Phys.**A18** (2003) 3921.
 - [69] LSND Collaboration, A. Aguilar *et al.*, Phys.Rev.**D64** (2001) 112007; hep-ex/0104
 - [70] MiniBooNE Collaboration W.C. Louis *et al.*, Proceeding of 10th International Workshop on Neutrino Telescopes, Venice, Italy, 11-14 Mar 2003 (vol. I, p. 181).